Boundary Conditions

As discussed before the particular solutions for flow problems using N-S equations require appropriate number of boundary conditions on the various flow variables.

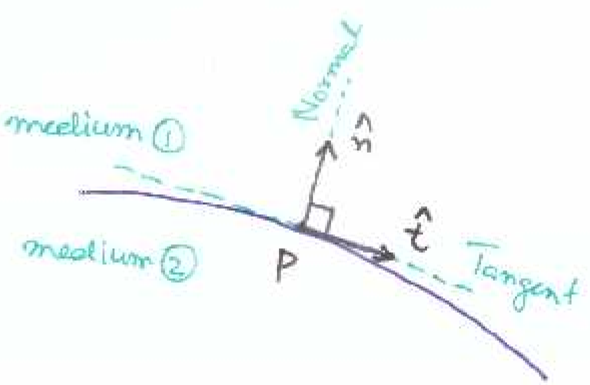
" The flow behaviour of the same fluid in different geometries is different because of different Boundary / Initial conditions."

The conditions at a material boundary or interface (solid-fluid interface, fluid-fluid interface) can be broadly classified as:

A: Kinematic conditions

B: Dynamic conditions.

Kinematic conditions:



At any point 'P' on the instantaneous interface between the two continous media, the following two conditions apply:

1. The local normal velocity is continous across the interface → No penetration condition

Mathematically,

$$(\vec{V})_1 \cdot \hat{n} = \qquad\qquad — B.1$$

$\vec{V}_I$: Velocity at interface

$(\vec{V})_2$, $(\vec{V})_1$: velocity in the neighborhood of interface on medium ② side & medium ① side

Physically the condition implies that particles of media on either side do not cross the interface.

2] The local tangential component of velocity is continous across the interface → No-slip condition

Mathematically

$$(\vec{V})_2 \cdot \hat{t} = \qquad\qquad — (B.2)$$

Note: In 3D, a surface/interface in general can be characterised locally by one normal and two tangential directions.

Thus in 3D, At any pt. on the interface we can write <u>one</u> no-penetration condition and <u>two</u> no-slip conditions.

Limitations:

1. The No-penetration condition is generally applicable to a Solid-Fluid Interface which is non-porous.

gas liquid

This condition must be used with care when a (Liquid-Liquid) Interface or a (Liquid-gaseous) interface is involved.

A liquid-liquid interface can only exist for immiscible pair of liquids. For such cases no-penetration can be used.

For a liquid-gaseous-interface, the no-penetration condition can be used only if the evaporation effect is negligible.

2. The no-slip condition is applicable to any type of S-F (solid) or F-F (Fluid) interface.

The no-slip condition is sometimes <u>wrongly</u> attributed to viscosity of fluid. Viscosity is in a way connected to ~~viscosity~~ no-slip condition.

An important Remark:

The No-penetration and No-slip conditions are two independent conditions. However, the two conditions when applicable to a S-F or F-F interface can be combined as follows:

$$(\vec{V})_1 \cdot \hat{n} =$$

$$\Rightarrow \quad \vec{V}_1 \cdot \hat{n} =$$

$$(\vec{V})_1 \cdot \hat{t} =$$

$$(\vec{V})_1 \cdot \hat{t} =$$

Since normal as well as tangential components are equal on either side of interface.

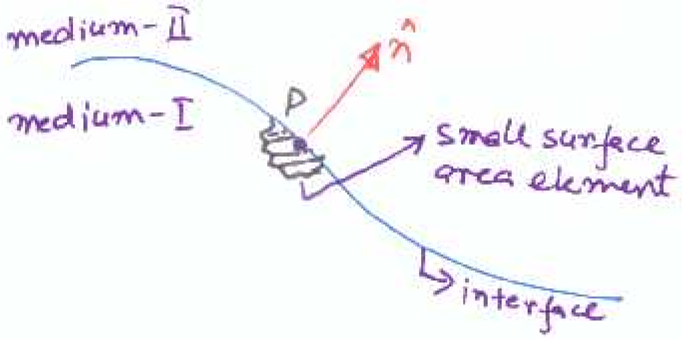
$$\Rightarrow \quad \vec{V}_1 = \vec{V}_2 \quad B \cdot 1 + B \cdot 2$$

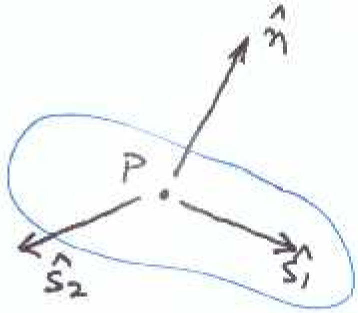
\# In some texts the last condition $\vec{V}_1 = \vec{V}_2$ is incorrectly mentioned as no slip condition.

\# The kinematic conditions B.1 & B.2 are valid in the continuum regime. ie the media on either side of the interface (S-F, F-F) can be treated as a continuum.

Dynamic conditions.

The dynamic conditions are relations between stress vector components (Pl. Note: We didn't mention stress tensor here. This is due to the fact that if we have a well defined area, with area normal, then stress has 3 components only and thus behaves like a vector. The interface is well defined by its normal and tangential directions) in the neighborhood of a point lying on the S-F or F-F interface, on either side of the interface.





$\hat{s}_1, \hat{s}_2$ are any two mutually orthogonal tangential unit vectors at point P on the elemental surface area dS.

The dynamic conditions can be expressed as:

a) Solid - fluid surface

All local stress vector components are continous across the surface.

b) Fluid- Fluid interface.

radii of curvature

For 3D, two radii of curvature are needed to define surface overall curvature

Surface tension

All local stress vector components are discontinous across the surface.

For a planar F-F interface.

$R_1 = R_2 = \infty$, Although S exists

$\Rightarrow (\sigma_{nn})_{I} = (\sigma_{nn})_{II} \Rightarrow$ Normal stresses are continous

If surface tension 'S' is negligible/or does not vary along the interface..

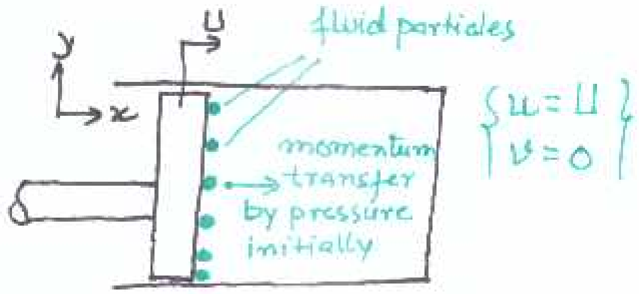
$$\Rightarrow \left.\begin{aligned} (\sigma_{ns_1})_{I} &= (\sigma_{ns_1})_{II} \\ (\sigma_{ns_2})_{I} &= (\sigma_{ns_2})_{II} \end{aligned}\right\}$$ Shear stress are also continous.

Summary:

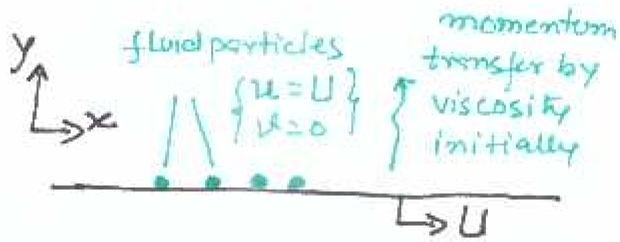
1. The presence of a physical interface of a fluid with a solid or another fluid is communicated through the kinematic and dynamic boundary conditions
2. The fluid motion is influenced by these boundary conditions.
3. For a rigid solid-fluid interface only the kinematic B.C's are relevant.

4. Generation of fluid motion

The kinematic boundary conditions can play a very important role in generation of fluid motion.



fluid motion caused by no-penetration condition.



fluid motion caused by no-slip condition.

Note: The no-slip condition transfers fluid motion to interior fluid particles by viscous effects.

\# No-slip condition is able to influence the fluid motion only though viscosity.

\# This implies that for inviscid flow analysis or models, the no-slip condition is not relevant.

Dimensionless formulation.

The Navier-Stokes equations derived earlier for an incompressible, homogenous, constant property flow contain dimensional quantities (t, x, y, z or any space coordinates), $(\vec{V}, p)$ and some dimensional parameters like (ρ_0, μ_0)

Therefore, we have relation(s) between several dimensional variables.

" Invoking Buckingham-π theorem, it is possible to reduce the parameters by combining them into dimensionless group(s) or parameters".

\# The approach adopted is slightly different to what was taught earlier in FM-I course.

\# This is because the viscous flow model (governing eqns and related boundary/initial conditions) was not developed earlier.

\# The basic idea in converting the N-S equations (or any other type of equation) into dimensionless form is to replace all dimensional variables (space coordinates and time included, as they are also variables) by corresponding dimensionless variables.

How this is achieved?

$$Q_{Dim} = \underbrace{(\text{Scale})}_{\substack{\text{Order of magnitude} \\ \text{expected for } Q_{Dim}}} \underbrace{Q^*}_{\text{Dimensionless}} = Q_s Q^*$$

For example, in a group of 60 students in a class, we consider the "Height of a student" say 'H' as a variable.

Then,

$$H = \underbrace{(\text{Scale})}_{\substack{\text{order of magnitude} \\ \approx (1m - 1.8m)}} H^*$$

$\longrightarrow$ Scale $= H_s = 1.5m$

Someone else can choose any other $1 < H_s < 1.8$

Now consider the N-S eqns,

The space coordinates are present in the del operator

$$\nabla \equiv \qquad \equiv \hat{e}_i \frac{\partial}{\underbrace{\partial s_i}_{}}$$

lengths of disp. vectors along different coordinate directions.

Steps involved in converting N-S eqns in dimensionless form.

1. Identify the variables — flow variables / space, time — and choose the corresponding scales.

$$(\text{space}\cdot\text{displ}) = L_s\,(\text{space-displ})^*$$

$$\text{time} = t = t_s t^*$$

$$\vec{V} = \qquad , \; p =$$

\# Notice that if instead of absolute pressure, we choose to work with gauge pressure $p_g = (p - p_o)$ | where p_o is any reference pressure . e.g. p_{atm}

the N-S eqn pressure force term remains unaffected

i.e $\nabla p = \nabla p_g$

(since p_o is a constant value).

Now, the gauge pressure can be converted into dimensionless form as,

$$p_g = p_s\, p_g^*$$ ← dimensionless gauge pr.

\# The use of gauge pressure is for ease and convenience in engineering practice.

\# For non-dimensional approach, we can work with abs. pr also.

Note: The parameters ρ_o, μ_o are not converted into dimensionless forms. We only need to convert the variables

2. Converting the equations

$\nabla \equiv$ $\equiv$ $\equiv$

$\nabla \equiv$

$\nabla^2 \equiv \nabla \cdot \nabla \equiv$

$\frac{\partial}{\partial t} \equiv$; $\vec{V} =$, $p = p_s p^*$

→ Continuity: $\nabla \cdot \vec{V} = 0$

→ Momentum Eqn:

$\rho_0 [$

ρ_0

Divide by $\rho_0 U_s^2 / L_s$

Let $\vec{g} = g\hat{i}_g$

Dimensionless parameters in momentum eqn:

1) $\Pi_1 =$

2) $\Pi_2 =$

3) $\Pi_3 =$

4) $\Pi_4 =$

Π_1 plays a role only when dealing with unsteady flows or performing a transient analysis of a steady flow: (time history of how fast the steady state is attained).

Flows are unsteady
- Unsteady forcing $[F_s \sin\omega t$ generally periodic having period $= T_f$
- Internal instabilities (Turbulent flows)

when the time scale 't_s' cannot be assessed,

1) During transient analysis of a steady flow, wherein we are not sure regarding the duration of transient phase.

2) Unsteady flow caused by internal instabilities.

$$t_s = L_s/U_s \equiv \text{Residence time scale} = t_R$$

$$\Pi_1 = \frac{L_s}{t_s U_s} = 1.0 \implies \Pi_1 \text{ is fixed at unity.}$$

For unsteady forcing, wherein time scale of the forcing function can be determined.

$$\Pi_1 = \qquad = \frac{t_R}{T_f}$$

$$\Pi_2 = \qquad = \frac{g}{U_s^2/L_s} = \frac{\text{gravitational accn}}{\text{order of fluid accn or inertia}}$$

This Π group is in the name of Froude and defined as

$$F_r =$$

$$\Rightarrow \quad \Pi_2 =$$

For flows having very high F_r, Π_2 becomes small

$\Rightarrow$ gravity terms in momentum eqn. $(= \Pi_2 \hat{i}_g)$ becomes small and can be neglected.

$\Pi_3 =$, for incompressible homogenous flow,

if we apply Bernoulli's theorem along a stream line then changes in pressure have an order in $\rho_0 U_s^2$

$$\Rightarrow p_s = \rho_0 U_s^2$$

$$\Rightarrow \Pi_3 = 1.0 \rightarrow \text{becomes fixed!}$$

$\pi_4 =$ $\rightarrow \pi_4 =$

$$= \frac{\text{order of viscous stresses}}{\text{order of fluid accn. or inertia}}$$

This π_4 group or parameter is related to a very celebrated / important dimensionless number for all viscous flows, introduced for the first time by Reynolds.

$$\text{Reynolds no: } Re = \frac{\rho_0 U_s L_s}{\mu_0}$$

$$\pi_4 = \frac{1}{Re}$$

As Re becomes small, the flow experiences large effects of viscosity and as Re becomes Large, the overall effects of Viscosity becomes small.

Note:

For any viscous flow, even if Re is very large (π_4 is very small), the viscous term $\equiv \frac{1}{Re} \nabla^{*2} \vec{V}^*$ cannot be dropped entirely, as it is containing the highest order (in space) derivatives of velocity $\Rightarrow$ B.C requirement

Summary:

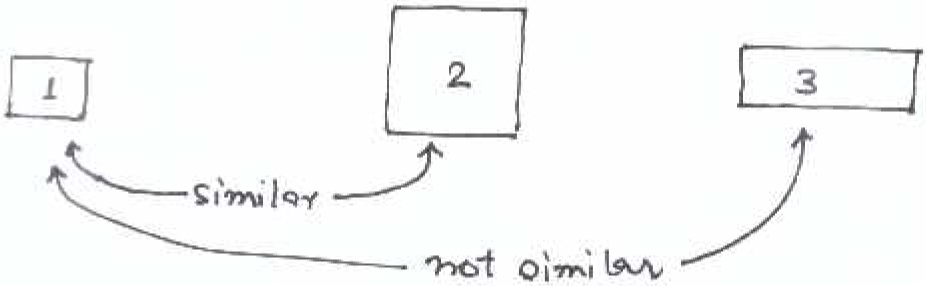
1. Dimensionless formulations directly generate the dimensionless Π-groups or numbers relevant for a given flow model. Its an alternative to Buckingham Π-theorem

Buckingham Π - theorem is useful if the governing equations themselves are not known.

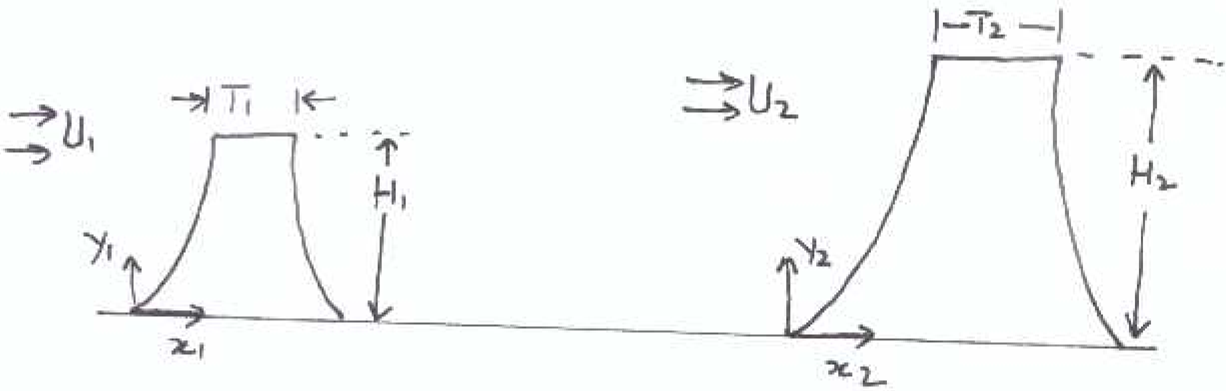
2. The relative importance of various terms in N-S eqns can be judged by the magnitude of the various Π-groups or dimensionless parameters involved.

Dynamic Similarity

We have the concept of geometric similarity. Two geometries are similar when the shapes are similar and the corresponding dimensions have the same ratio.



In fluid flow problems also we can have geometric similarity.



Steady flow past a cooling tower:

$\frac{T_2}{T_1} = \frac{H_2}{H_1}$, side geometry is described by the same type of function

Stationary fixed cylinders

Steady, 2D flow past a circular cylinder.

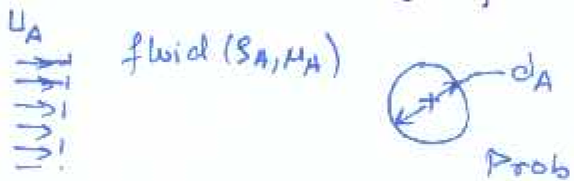
In the above examples apart from geometric similarity, there is similarity of the velocity boundary conditions (No-slip + No-penetration S-F interfaces and same types of free-stream conditions). Such a similarity is termed as Kinematic similarity. When working with unsteady flows, Kinematic similarity also includes initial conditions.

Two flows are said to be 'Dynamically similar' when the following conditions hold:

1) Two flows are geometrically and kinematically similar.
2. The two flows have the same dimensionless solution when their dimensionless solutions are obtained in the dimensionless domains.

Next we will examine the conditions that can lead to dynamic similarity between two geometrically and kinematically similar flows.

Consider an incomp., hom., constt. property flow of two different fluids around two spherical objects of different size. The objects are immersed in a large expense of fluid



Prob A

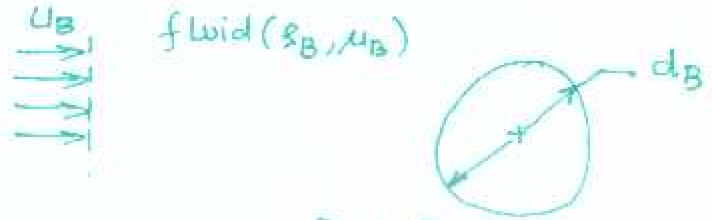
$L_s = d_A$, $U_s = U_A$, $p_s = \rho_A U_A^2$, $t_s = d_A/U_A = t_R$

$\Downarrow$ $(\pi_1 = \pi_3 = 1.0)$

$$\nabla^* \cdot \vec{V}_A^* = 0$$

$$\frac{D\vec{V}_A^*}{Dt^*} = -\nabla^* p_{mA}^* + \frac{1}{Re_A} \nabla^{*2} \vec{V}_A^*$$

$$Re_A = \frac{\rho_A U_A d_A}{\mu_A}$$



Prob B

$L_s = d_B$, $U_s = U_B$, $p_s = \rho_B U_B^2$, $t_s = d_B/U_B = t_R$

$\Downarrow$ $(\pi_1 = \pi_3 = 1.0)$

$$\nabla^* \cdot \vec{V}_B^* = 0$$

$$\frac{D\vec{V}_B^*}{Dt^*} = -\nabla^* p_{mB}^* + \frac{1}{Re_B} \nabla^{*2} \vec{V}_B^*$$

$$Re_B = \frac{\rho_B U_B d_B}{\mu_B}$$

'Since Re is the only dimensionless parameter for this problem, the dimensionless solutions would depend on Re only'.

If $Re_A = Re_B$

we can conclude,

$$\left.\begin{aligned} \vec{V}_A^*(\vec{r}^*, t^*) &= \vec{V}_B^*(\vec{r}^*, t^*) \\ p_{mA}^*(\vec{r}^*, t^*) &= p_{mB}^*(\vec{r}^*, t^*) \end{aligned}\right\} \text{Dynamic similarity}$$

What has been illustrated by the ~~above~~ previous example can be summarised as follows:

Two geometrically, kinematically, similar flows are dynamically similar if all the dimensionless parameters or π-groups relevant in the dimensionless formulations (Equations, + B.C + I.C) of the two problems are made identical (values are kept same).

Finally, we examine the relations between the dimensional solutions of the two problems under the condition of Dynamic similarity.

$$\vec{V}_A^* = \qquad \text{at some } (\vec{r}^*, t^*)$$

$$\Rightarrow \frac{\vec{V}_A(\vec{r}_A, t_A)}{U_A} =$$

where $\dfrac{\vec{r}_A}{d_A} =$ $\Rightarrow$ corresponding points

$$\frac{t_A U_A}{d_A} = \qquad \Rightarrow \text{corresponding time instants}$$

Using Dynamic Similarity!

Dynamic similarity paves the way and forms the basis of carrying out scaled up/down experiments of real world flows/flows in various engineering applications.

For example, it is possible to test scaled down models of airplane/aircraft components/missiles etc. in a suitably designed laboratory experimental apparatus.

Even in carrying out computer simulations of flows, the idea of matching the π groups or dimensionless parameters to achieve dynamic similarity is exploited.

Summary:

1. Two flows are dynamically similar if:
   a) They possess geometric + kinematic similarity (velo. B.C & I.C)
   b) All the dimensionless parameters relevant to the problem for the two cases must match in their respective magnitudes.
2. Under the condition of dynamic similarity, the flow field of the two flows can be related to each other. Thus if one flow field is known or determined, the other can be predicted using the above relation(s).
3. Dynamic similarity forms the basis of studying/simulating many real world flows through scaled up/down physical experiments in laboratories and even in virtual experiments on computers.